

The relic abundance of long-lived heavy colored particles

Junhai Kang,^a Markus A. Luty^a and Salah Nasri^{ab}

^a*Physics Department, University of Maryland,
College Park, Maryland, U.S.A.*

^b*Department of Physics, University of Florida,
Gainesville, Florida, U.S.A.*

E-mail: junhai2000@gmail.com, luty@physics.ucdavis.edu,
nasri.salah@gmail.com

ABSTRACT: Long-lived colored particles with masses $m \gtrsim 200$ GeV are allowed by current accelerator searches, and are predicted by a number of scenarios for physics beyond the standard model. We argue that such “heavy partons” effectively have a geometrical cross section (of order 10 mb) for annihilation at temperatures below the QCD deconfinement transition. The annihilation process involves the formation of an intermediate bound state of two heavy partons with large orbital angular momentum. The bound state subsequently decays by losing energy and angular momentum to photon or pion emission, followed by annihilation of the heavy partons. This decay occurs before nucleosynthesis for $m \lesssim 10^{11}$ GeV for electrically charged partons and $m \lesssim$ TeV for electrically neutral partons. This implies that heavy parton lifetimes as long as 10^{14} sec are allowed even for heavy partons with $m \sim$ TeV decaying to photons or hadrons with significant branching fraction.

KEYWORDS: Cosmology of Theories beyond the SM, QCD.

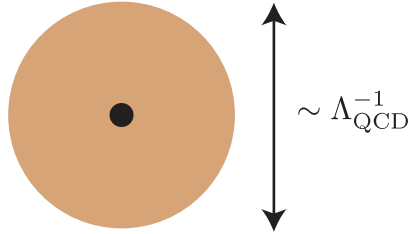


Figure 1: A schematic representation of a hadron made of a heavy parton and surrounded by QCD “brown muck.”

The phenomenology and cosmology of heavy ($m \gg \text{GeV}$) long-lived colored particles has received renewed attention recently because of the proposal of “split supersymmetry” [1]. Another possible motivation is having a long lived gluino [2] or squark [3] as the next-to lightest superpartner in weak scale supersymmetry. More generally, it is a phenomenologically interesting possibility that such particles could exist. For $m \gtrsim 200 \text{ GeV}$ such particles are consistent with current collider bounds, and if $m \lesssim 2 \text{ TeV}$ they will be accessible at LHC [4].

This note concerns the cosmology of such particles, which we refer to generically as “heavy partons,” since they are constituents of exotic long-lived hadrons at low energies. This has been studied by a number of authors, and there is some controversy about the the correct relic abundance, even at the level of the order of magnitude [5]. The disagreement is over the extent to which the strong interactions enhance the annihilation cross section in the early universe over the perturbative value. At temperatures below the deconfinement temperature $T_c \simeq 180 \text{ MeV}$, the heavy partons are confined inside hadrons. Just as in heavy quark effective theory [6] it is useful to picture these hadrons as consisting of a heavy parton surrounded by QCD “brown muck” with a radius of order $R_{\text{had}} \sim \text{GeV}^{-1}$ (see figure 1). If the heavy parton is a color triplet, the brown muck will involve at least one light quark, while if the heavy parton is a color octet, it may involve just gluons. Our arguments will not depend on the details of the brown muck.

We therefore consider two heavy hadrons in the early universe at temperatures below the QCD phase transition. It is clear that the strong interactions give a geometrical cross section (of order R_{had}^2) for the heavy hadrons to *interact*, but this only means that the brown muck of one hadron interacts with that of the other. In order for the partons to *annihilate*, the wavefunctions of the heavy partons themselves must overlap, so the direct annihilation cross section is proportional to m^{-2} .

We will argue that the strong interactions nonetheless give rise to an effective geometrical cross section for annihilation at temperatures below the QCD phase transition. The first stage of this process is illustrated in figure 2. This is a capture process in which two heavy hadrons interact to form a bound state, with the energy carried off e.g. by a pion. As we will see, this bound state has large orbital angular momentum ($L \sim 10$ for $m \sim \text{TeV}$) and therefore does not decay promptly. In the second stage, the bound state loses energy and angular momentum, e.g. by emitting pions and/or photons. Bound states

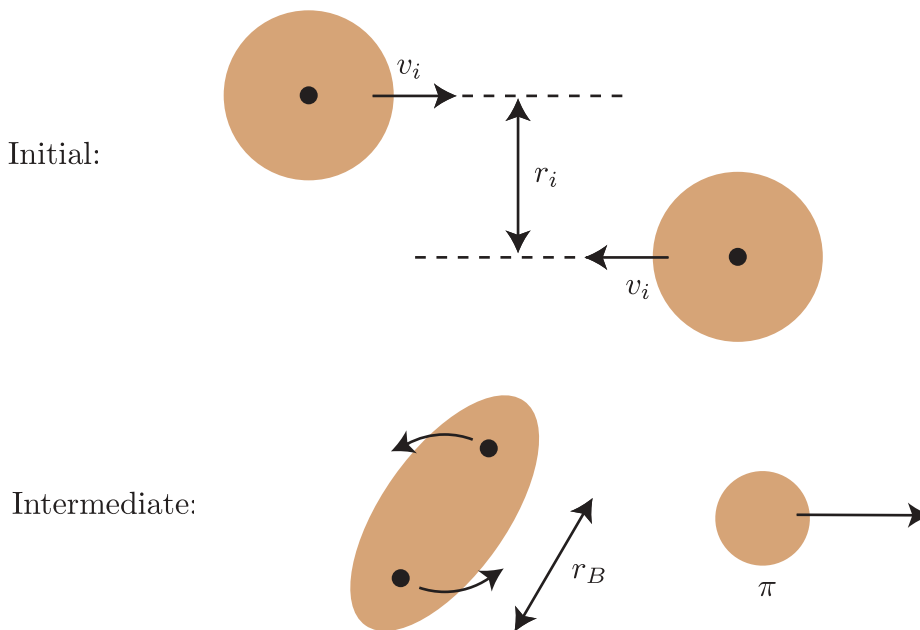


Figure 2: The formation of a highly excited bound state.

with binding energy larger than the temperature T survive collisions with particles in the thermal bath, and eventually decay through annihilation. The net result is that the heavy partons effectively have an enhanced cross section for annihilation given by the capture cross section. (This is similar to the scenario originally described in [1] for stable gluinos.) The phenomenological implications of this result will be discussed at the end of this note.

We now explain why the capture process is unsuppressed. The first point is the existence of the bound state. The potential between two heavy partons can be written schematically as a sum of a short-distance Coulomb interaction and an attractive linear term representing the effects of confinement:

$$V(r) \sim \frac{C\alpha_{\text{QCD}}}{r} - \sigma r. \tag{1}$$

Here $\sigma \sim \Lambda_{\text{had}}^2$ is the string tension, and C is a group theory factor that depends on the color representation of the heavy partons. The color Coulomb force is always attractive in the color singlet channel (i.e. $C < 0$). We also expect the long-range part of the potential to be attractive in the color singlet channel, since it is responsible for color confinement. The energy spectrum of the system of two heavy partons therefore looks as shown schematically in figure 3. The low-lying states are Coulombic, with energy splittings of order $\alpha_{\text{QCD}}^2 m \gg \Lambda_{\text{had}} \sim \text{GeV}$, while the states near the continuum threshold are dominated by the linear term. Spin-dependent interactions are suppressed by $1/m$, so spin excitations are small and we do not expect them to play an important role in the process we are considering. The rotational excitations are very important for determining the properties of the intermediate bound state. The minimum radius for a given angular momentum is determined by the

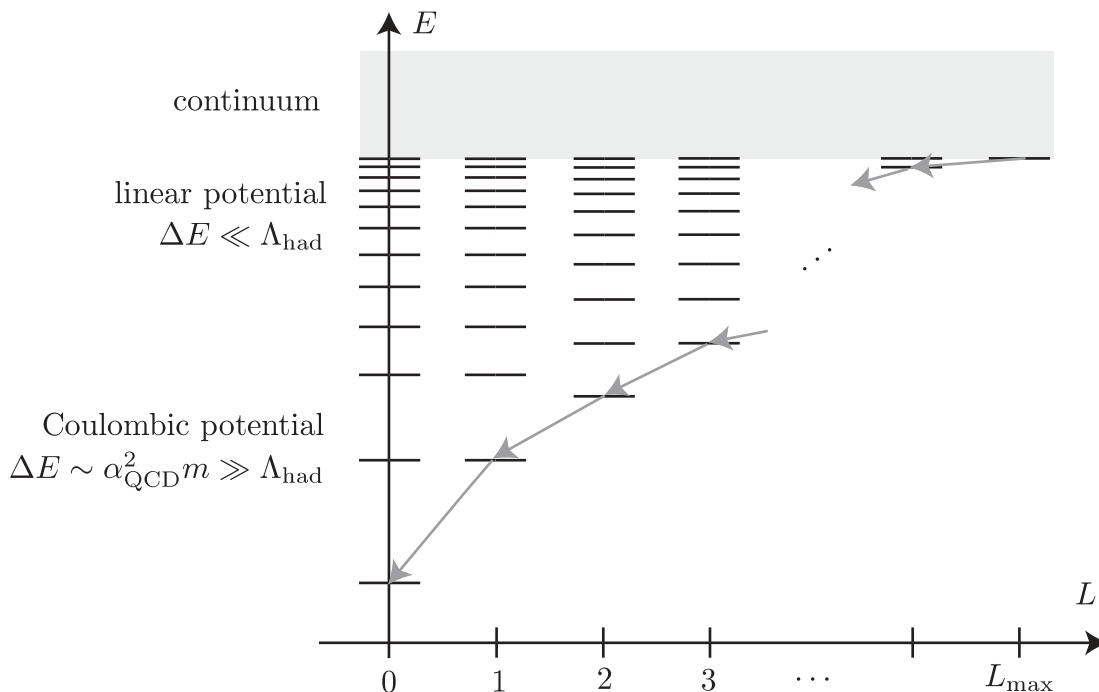


Figure 3: Spectrum of the system consisting of two heavy partons. (Spin excitations are not shown.) Bound states with large angular momentum will decay through a series of $\Delta L = 1$ transitions, as shown.

circular orbits. In the regime where the linear term dominates, we have

$$r_{\min} \sim \left(\frac{L^2}{\sigma m} \right)^{1/3}. \quad (2)$$

The largest angular momentum occurs for $r \sim R_{\text{had}}$. At larger radii, the string confining the heavy partons will break and the spectrum becomes a continuum of two-particle states.¹ The largest angular momentum of a bound state is therefore

$$L_{\max} \sim \left(\frac{m}{\Lambda_{\text{had}}} \right)^{1/2} \sim 30 \left(\frac{m}{\text{TeV}} \right)^{1/2}. \quad (3)$$

This estimate is consistent with the fact that there are expected to be $L = 3$ states in the the Υ system below the $B\text{-}\bar{B}$ threshold [7]. This gives $L_{\max} \sim 3(m/m_b)^{1/2} \sim 40$ for $m \sim \text{TeV}$.

We now consider the process depicted in figure 2, taking place at temperatures $T \lesssim T_c$. We are interested in impact parameters $r_i \lesssim \Lambda_{\text{had}}^{-1}$, so that the brown muck clouds overlap, and therefore interact strongly. (We neglect the possibility that pion exchange gives a longer range attractive interaction.) To understand the possible transition to a bound state, we must understand what quantities are conserved in this transition, taking into account the special kinematics of the situation. Unlike a weakly interacting relic, a colored

¹String breaking is suppressed for large N_c . We will not keep track of factors of N_c in this paper.

particle will be in kinetic equilibrium with gluons at the deconfinement temperature. The initial velocity is therefore determined by the temperature to be

$$v_i \sim \left(\frac{T_B}{m}\right)^{1/2}, \quad (4)$$

where T_B is the temperature where the bound state is formed. Note that the momentum is large compared to the inverse range of the interaction

$$p_i \sim mv_i \sim (mT_B)^{1/2} \gg \frac{1}{R_{\text{had}}}, \quad (5)$$

so the scattering process is not dominated by s -wave scattering, and we expect higher partial waves to be important. On the other hand, the velocity is slow enough that the forces exerted by the brown muck can change the velocity significantly:

$$\frac{\Delta v}{v_i} \sim \frac{a\Delta t}{v_i} \sim \frac{(\Lambda_{\text{had}}^2/m)(R_{\text{had}}/v_i)}{v_i} \sim \frac{\Lambda_{\text{had}}}{T_B} \gtrsim 1. \quad (6)$$

We see that the velocity is not conserved in the collision, even though the partons are heavy.

Energy and angular momentum must of course be conserved in the collision. Energy conservation is satisfied by the emission of a pion (or perhaps several) that carries away the binding energy. The pion can also carry away some angular momentum, but only $\Delta L \sim \text{few}$. The typical initial angular momentum is

$$L_i \sim mv_i r_i \sim (mT_B)^{1/2} R_{\text{had}} \sim 10 \left(\frac{m}{\text{TeV}}\right)^{1/2} \left(\frac{T_B}{T_c}\right)^{1/2}. \quad (7)$$

The initial angular momentum is large, but note that L_i becomes smaller than L_{max} for $T_B < T_c$. This means that the binding energy is

$$B = E_{\text{max}} - E_f \sim \left(\frac{\sigma^2 L_{\text{max}}^2}{m}\right)^{1/3} - \left(\frac{\sigma^2 L_i^2}{m}\right)^{1/3} \sim \left(\frac{\sigma^2 L_{\text{max}}^2}{m}\right)^{1/3} \sim \Lambda_{\text{had}}. \quad (8)$$

The initial kinetic energy is only $T_c \sim m_\pi$, but the binding energy is sufficiently large to produce a pion.

It is also easy to see that the transverse distance between the heavy partons is allowed to change significantly during the collision. The forces of the brown muck give

$$\Delta r_\perp \sim a_\perp \Delta t^2 \sim \frac{\Lambda_{\text{had}}^2}{m} \left(\frac{R_{\text{had}}}{v}\right)^2 \sim \frac{1}{T_B} \gtrsim R_{\text{had}}, \quad (9)$$

where \perp refers to the component perpendicular to the line connecting the heavy partons. Conservation of angular momentum gives

$$\frac{\Delta r_\perp}{r} \sim \frac{\Delta v_\perp}{v} \gtrsim 1. \quad (10)$$

Based on the considerations above, we argue that the cross section for the formation of the bound state is geometrical, i.e.

$$\sigma_{\text{form}} \sim \pi R_{\text{had}}^2. \quad (11)$$

The reason is simply that there is no symmetry or kinematic factor that suppresses the transition, so it should proceed at the going rate for strong interactions. In fact, it is possible that longer range forces due to pion exchange increase the cross section, a possibility that we will neglect here. As we have seen, the transition can occur for impact parameters of order R_{had} without violating any exact or approximate conservation law. Furthermore, the energy of the emitted pion(s) is of order Λ_{had} , so there is no low-energy suppression.

It is important to keep in mind that the geometrical cross section gives the rate for the bound state to *form*, and we must consider the subsequent evolution of the bound state to see whether it enhances the *annihilation* of the heavy partons themselves. The first issue is whether bound states of the kind we are discussing can survive in the thermal bath long enough so that it can radiate away energy and angular momentum and eventually annihilate. The biggest threat to their survival is collisions with photons, since pions have low number density for $T < m_\pi \sim T_c$. States with binding energy $B > T$ cannot be destroyed efficiently, since the probability for finding a photon in the thermal bath with energy B is suppressed by $e^{-B/T}$, which drops rapidly for $T < B$. (This rate will have additional suppression if the heavy partons and the brown muck are electrically neutral.) To find the precise value of the temperature below which destruction becomes inefficient requires a detailed hadronic model, but it is of order T_c .

One might also worry about successive collisions with photons gradually increasing the energy of the bound state until it becomes unbound. The dominant process is inverse decay $B\gamma \rightarrow B'$, where B and B' are bound states. However, these merely establish an equilibrium distribution of excitations (in which bound states are more numerous) on a time scale given by the rate for the decay $B' \rightarrow B\gamma$.

We now consider the decay of the bound state. We consider first the case of electrically charged heavy partons, which can decay by the familiar process of photon radiation. The bound state decays by first radiating away energy and angular momentum to get to a bound state with $L \sim 1$, which then decays by annihilation into quarks or gluons. We first estimate the decay rate to an $L \sim 1$ state. The linear term dominates the force on a heavy parton for $r \gtrsim R_c$, where

$$R_c \sim \left(\frac{\alpha_{\text{QCD}}}{\sigma} \right)^{1/2}. \tag{12}$$

This corresponds to a binding energy

$$E_c \sim \frac{\alpha_{\text{QCD}}}{R_c} \sim (\alpha_{\text{QCD}}\sigma)^{1/2}. \tag{13}$$

The time to lose energy of order E_c can be estimated by using the Larmor formula $\dot{E} \sim \alpha a^2$, where $a \sim \sigma/m$ is the acceleration in the linear potential. Since the acceleration is constant, we have

$$\tau(\Delta E \sim -E_c) \sim \frac{E_c}{\dot{E}} \sim \frac{\alpha_{\text{QCD}}^{1/2} m^2}{\alpha \Lambda_{\text{had}}^3}. \tag{14}$$

The time to lose the remaining energy can again be estimated by using the Larmor formula, with a determined from the Coulomb potential. It is easy to check that this is dominated

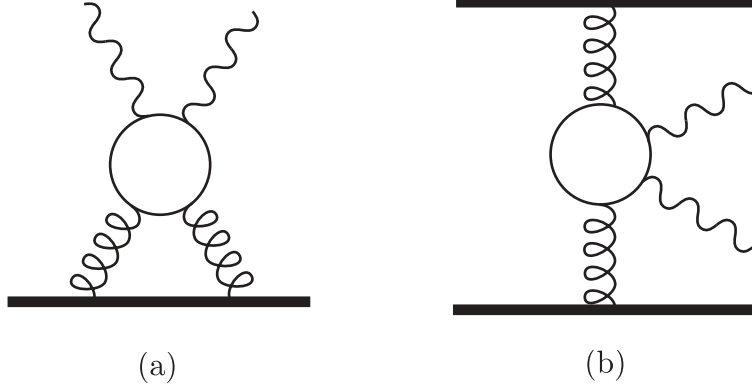


Figure 4: Diagrams contributing to the decay $B' \rightarrow B\gamma\gamma$. The thick lines are heavy partons and the thin lines are light quarks.

by the energy loss for binding energies of order E_c , and we get the same estimate eq. (14) for this decay time. The subsequent annihilation is much more rapid, as can be seen for example from the formula for the annihilation rate for an $L = 0$ state:

$$\Gamma_{\text{annihilation}} \sim \frac{4\pi\alpha_{\text{QCD}}^2}{m^2} |\psi(0)|^2, \quad (15)$$

where $\psi(0)$ is the radial wavefunction of the bound state evaluated at the origin. Since $\psi(0) \sim (\alpha_{\text{QCD}}m)^{3/2}$ for the ground state, we have $\Gamma_{\text{annihilation}} \sim 4\pi\alpha_{\text{QCD}}^5 m$. We conclude that the decay rate for the bound state is given by eq. (14). This decay occurs before nucleosynthesis ($\tau \lesssim 1$ sec) for $m \lesssim 10^{11}$ GeV, where we have used $\Lambda_{\text{had}} \sim \text{GeV}$. For larger masses, the late decays to photons will affect nucleosynthesis, and the model is ruled out [8].

We now discuss the decays of the bound state in the case where the heavy partons are electrically neutral. We assume that the brown muck is also electrically neutral. The energy can then be carried away by either photons or pions. The photon rate is a loop effect, and is suppressed by the small electromagnetic coupling, but the pion rate is potentially kinematically suppressed by the pion mass, which can be larger than the energy differences between states with $\Delta L \sim 1$ for large m . We will confine our attention to the two photon decays, since we will see that they are sufficiently rapid for the most interesting range of m .

Two photon decays occur via diagrams such as the ones in figure 4. Since we are interested in energy and momentum transfers small compared to Λ_{had} , we can parameterize the effects of diagrams such as the one in figure 4a by effective operators in the heavy parton effective theory:

$$\mathcal{L}_{\text{int}} \sim \frac{e^2}{\Lambda_{\text{had}}^3} F^{\mu\rho} F^\nu{}_\rho \psi^\dagger \left(v_\mu + \frac{iD_\mu}{m} \right) \left(v_\nu + \frac{iD_\nu}{m} \right) \psi \sim \frac{e^2}{\Lambda_{\text{had}}^3} F^{0i} F^{ij} \psi^\dagger \frac{i\partial_j}{m} \psi + \dots \quad (16)$$

Here ψ is a heavy parton field of dimension $\frac{3}{2}$ and v_μ is the 4-velocity of the heavy parton. We require two photon fields by charge conjugation invariance, and electromagnetic gauge invariance then forces this to be proportional to the field strength tensor. Since we are interested in orbital transitions of the heavy parton, we want a term with at least one

spatial derivative acting on the heavy parton field. The factor of $1/m$ arises because the heavy parton effective theory can depend on the 4-velocity and the residual momentum only in the combination $p_\mu = mv_\mu + k_\mu$ [9]. (We ignore the spin indices, which are not relevant for estimating the rate for orbital transitions.) Note that derivatives acting on the photon fields give powers of $E_\gamma \sim \Delta E \ll \Lambda_{\text{had}}$, and so the expansion in terms of derivatives acting on the photon fields should be valid. Derivatives acting on the heavy parton fields give powers of $1/r_B$, where r_B is the size of the bound state. Since $r_B \gg 1/m$, these should also be suppressed. There are also 4-parton processes such as the one shown in figure 4b that give rise to 4-parton operators that are nonlocal on the scale R_{had} . These cannot be treated as local in the effective field theory we are considering, and it is more subtle to do the power counting for these operators. The operator eq. (16) can be treated using standard quantum mechanics techniques, and we are confident that it correctly counts the powers of m , Λ_{had} , and E_γ in the process. We will therefore use it to get at least an upper bound on the lifetime of the bound state.

Integrating over phase space, we obtain

$$\Gamma(B' \rightarrow B\gamma\gamma) \sim \frac{\alpha^2 E_\gamma^7}{4\pi m^2 \Lambda_{\text{had}}^6 r_B^2}, \quad (17)$$

where r_B is the size of the bound state. The energy of a bound state in the linear regime is

$$E \sim \left(\frac{\sigma^2 L^2}{m} \right)^{1/3}, \quad (18)$$

where E goes from $E_{\text{min}} \sim \sigma R_c$ (where the potential becomes Coulombic, see eq. (12)) to $E_{\text{max}} \sim \Lambda_{\text{had}}$ (where the system becomes unbound). In terms of this energy variable, the energy difference between $\Delta L = 1$ states is

$$\Delta E \sim \left(\frac{\sigma^2}{mE} \right)^{1/2}. \quad (19)$$

Using $E_\gamma \sim \Delta E$, we can write the rate of energy loss as

$$\dot{E} \sim \Gamma \Delta E \sim \frac{\alpha^2 \Lambda_{\text{had}}^{14}}{4\pi m^6 E^6}. \quad (20)$$

The lifetime is therefore

$$\begin{aligned} \tau &\sim \int \frac{dE}{\dot{E}} \sim \int_{E_c}^{E_B} dE \frac{4\pi m^6 E^6}{\alpha^2 \Lambda_{\text{had}}^{14}} \\ &\sim \frac{4\pi m^6 E_B^7}{\alpha^2 \Lambda_{\text{had}}^{14}} \sim \frac{4\pi m^6}{\alpha^2 \Lambda_{\text{had}}^7} \left(\frac{T_B}{\Lambda_{\text{had}}} \right)^{7/3}, \end{aligned} \quad (21)$$

where we have used

$$E_B \sim \left(\frac{\sigma^2 L_i}{m} \right)^{1/3} \sim (\Lambda_{\text{had}}^2 T_B)^{1/3}. \quad (22)$$

Note that the lifetime is dominated by the bound states closest to threshold, where $\Delta E \ll \Lambda_{\text{had}}$ and the expansion above is valid. This decay occurs before nucleosynthesis for $m \lesssim 2.5$ TeV, where we use $T_B \sim 200$ MeV and $\Lambda_{\text{had}} \sim \text{GeV}$ to obtain the bound. (Note that since the rate is proportional to m^6 , the value of m is actually quite well determined.) It is possible that the decay rate to pions is more rapid, but we will not attempt to estimate it here.

Let us comment on the theoretical uncertainties in our analysis above. There are many numerical factors that we have estimated to be of order 1, and it is certainly possible that some of our estimates are numerically inaccurate. However, since we are interested in setting cosmological *limits*, the most conservative assumptions are those that weaken the limits. In the analysis above, we have attempted to make “middle of the road” estimates for all quantities. To strengthen the cosmological limits on heavy stable partons, one would have to demonstrate that the estimates above are incorrect, taking into account the large uncertainty in hadronic quantities. Taking this into account, we believe that the bounds we obtain are robust.

We now discuss the relic abundance of the heavy partons. At a temperature of order $T \sim m/30$ perturbative annihilation of heavy partons due to perturbative QCD gives a relic abundance $Y = n_P/s \sim 10^{-14}$ for temperatures $T \lesssim m/30$. Below the QCD phase transition, the second stage of annihilation described above further reduces the relic abundance and determines the final relic abundance. For $T = T_B \lesssim T_c$ the thermally averaged rate for annihilations (more precisely, formation of bound states that later decay) is

$$\langle \sigma |v| \rangle = \pi R^2 \left(\frac{T_B}{m} \right)^{1/2}, \tag{23}$$

where we expect $R \sim R_{\text{had}}$. This reduces the relic abundance until the annihilation rate drops below the Hubble expansion rate: $\Gamma = n_P \langle \sigma |v| \rangle \lesssim H \sim g_*^{1/2} T^2 / M_{\text{P}}$. Saturating this inequality gives a relic abundance of unbound partons of

$$Y_P = \frac{n_P}{s} \sim 10^{-18} \left(\frac{R}{\text{GeV}^{-1}} \right)^{-2} \left(\frac{T_B}{180 \text{ MeV}} \right)^{-3/2} \left(\frac{m}{\text{TeV}} \right)^{1/2}, \tag{24}$$

where $s = 2\pi^2 g_* T^3 / 45$ is the entropy density, and we use $g_* \simeq 10$ just below the QCD phase transition. We are neglecting entropy production at the QCD phase transition.

This is a very interesting relic abundance for the phenomenologically relevant mass range $m \lesssim \text{TeV}$. The cosmological bounds depend on the lifetime and decay modes of the heavy partons. If the lifetime is in the range of 10^2 sec to 10^6 sec, there are bounds arising from the fact that photon or hadronic decay products can affect nucleosynthesis [8]. For lifetimes in the range 10^6 sec to 10^{12} sec, there are bounds coming from the photodissociation of light elements [10]. For lifetimes in the range 10^6 sec to 10^{13} sec, there are bounds from distortions of the cosmic microwave background [11]. These bounds apply because we expect the decaying parton to have a significant branching ratio into both hadrons and photons. Remarkably, all of these bounds become ineffective for $m Y_P \lesssim 10^{-14} \text{ GeV}$, near the limit of our estimated relic abundance for the phenomenologically relevant range of masses

$m \lesssim \text{TeV}$. The conservative conclusion is therefore that such heavy partons are not excluded. For lifetimes in the range 10^{14} sec to 10^{18} sec, there are bounds from observations of the diffuse photon background [12]. These require that the lifetime of the partons is shorter than about 10^{14} sec if there is a significant branching fraction into photons or hadrons. Stable partons are ruled out by searches for heavy hydrogen [13]. Thus, the reduction in the relic density by several orders of magnitude over the perturbative prediction lengthens the allowed lifetime for TeV scale heavy partons by roughly 12 orders of magnitude!

Acknowledgments

We thank P. Bedaque, T. Cohen, T. Okui, and J. Wacker for discussions. This work is supported by the National Science Foundation grant no. PHY-0354401 and the University of Maryland Center for Particle and String Theory.

References

- [1] N. Arkani-Hamed and S. Dimopoulos, *Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC*, *JHEP* **06** (2005) 073 [[hep-th/0405159](#)].
- [2] S. Raby, *Gauge-mediated SUSY breaking with a gluino LSP*, *Phys. Lett.* **B 422** (1998) 158 [[hep-ph/9712254](#)].
- [3] U. Sarid and S.D. Thomas, *Mesino-antimesino oscillations*, *Phys. Rev. Lett.* **85** (2000) 1178 [[hep-ph/9909349](#)].
- [4] W. Kilian, T. Plehn, P. Richardson and E. Schmidt, *Split supersymmetry at colliders*, *Eur. Phys. J.* **C 39** (2005) 229 [[hep-ph/0408088](#)];
J.L. Hewett, B. Lillie, M. Masip and T.G. Rizzo, *Signatures of long-lived gluinos in split supersymmetry*, *JHEP* **09** (2004) 070 [[hep-ph/0408248](#)];
For a review see M. Fairbairn et al., *Stable massive particles at colliders*, *Phys. Rept.* **438** (2007) 1 [[hep-ph/0611040](#)].
- [5] S. Wolfram, *Abundances of stable particles produced in the early universe*, *Phys. Lett.* **B 82** (1979) 65;
C.B. Dover, T.K. Gaisser and G. Steigman, *Cosmological constraints on new stable hadrons*, *Phys. Rev. Lett.* **42** (1979) 1117;
R.N. Mohapatra and S. Nussinov, *Possible manifestation of heavy stable colored particles in cosmology and cosmic rays*, *Phys. Rev.* **D 57** (1998) 1940 [[hep-ph/9708497](#)];
H. Baer, K.-M. Cheung and J.F. Gunion, *A heavy gluino as the lightest supersymmetric particle*, *Phys. Rev.* **D 59** (1999) 075002 [[hep-ph/9806361](#)];
A. Arvanitaki, C. Davis, P.W. Graham, A. Pierce and J.G. Wacker, *Limits on split supersymmetry from gluino cosmology*, *Phys. Rev.* **D 72** (2005) 075011 [[hep-ph/0504210](#)].
- [6] For a review and original references see e.g. M.B. Wise, *Heavy quark theory*, [hep-ph/9411264](#).
- [7] See e.g. A. Grant and J.L. Rosner, *Dipole transition matrix elements for systems with power law potentials*, *Phys. Rev.* **D 46** (1992) 3862.
- [8] M. Kawasaki, K. Kohri and T. Moroi, *Big-bang nucleosynthesis and hadronic decay of long-lived massive particles*, *Phys. Rev.* **D 71** (2005) 083502 [[astro-ph/0408426](#)].

- [9] M.E. Luke and A.V. Manohar, *Reparametrization invariance constraints on heavy particle effective field theories*, *Phys. Lett.* **B 286** (1992) 348 [[hep-ph/9205228](#)].
- [10] M. Kawasaki and T. Moroi, *Electromagnetic cascade in the early universe and its application to the big bang nucleosynthesis*, *Astrophys. J.* **452** (1995) 506 [[astro-ph/9412055](#)].
- [11] W. Hu and J. Silk, *Thermalization constraints and spectral distortions for massive unstable relic particles*, *Phys. Rev. Lett.* **70** (1993) 2661; *Thermalization and spectral distortions of the cosmic background radiation*, *Phys. Rev.* **D 48** (1993) 485.
- [12] G.D. Kribs and I.Z. Rothstein, *Bounds on long-lived relics from diffuse gamma ray observations*, *Phys. Rev.* **D 55** (1997) 4435 [*Erratum ibid.* **D 56** (1997) 1822] [[hep-ph/9610468](#)].
- [13] T.K. Hemmick et al., *Search for low-Z nuclei containing massive stable particles*, *Phys. Rev.* **D 41** (1990) 2074;
T. Yamagata, Y. Takamori and H. Utsunomiya, *Search for anomalously heavy hydrogen in deep sea water at 4000 m*, *Phys. Rev.* **D 47** (1993) 1231.